Application of the numerical method for the propagation of distributions to the calculation of coverage intervals in the thermovision measurements

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1. Introduction

In the contemporary measurement theory, the new methods of accuracy determination are still investigated. It is possible, that the theory of uncertainty is the universal tool for estimate of the accuracy [1, 2]. This theory allows for the analysis of random interactions, even if the measurement model is strongly nonlinear. According to the theory of uncertainty, the standard uncertainty plays main role in the accuracy determination.

The problem of the determination of uncertainty components in thermowision measurement was described in [3-6]. In this paper the analysis of the accuracy of measurement method, based on the coverage interval idea was presented. In the simulation research, the model described in [4, 5] was assumed. It can be formulated as a function of five input variables:

$$T_{ob} = f\left(\varepsilon_{ob}, T_{atm}, T_{0,} \omega, d\right), \tag{1}$$

where: ε_{ob} – object emissivity, T_{atm} – temperature of atmosphere, K, T_0 – ambient temperature, K ω – relative humidity, *d* – distance between infrared camera and the object.

In the research of components of combined standard uncertainty, by means model (1), the Monte Carlo simulations were used. The expanded uncertainty, with the assumed level of confidence was calculated using the method for the propagation of distributions.

2. The method for the propagation of distributions and Monte Carlo simulations

The goal of the use the propagation of distributions is the determination of uncertainty, using Monte Carlo simulations. The fundamental purpose of computational procedure is to obtain statistical coverage interval of the measurement model output variable. It is necessary to emphasize, that the results of calculations are correct, even if the model is strongly nonlinear and the probability density functions of output are asymmetric. This situation takes a place in the thermovision measurements. The method for the propagation of distributions consists of following steps [7]:

- Determination of the output random variable of the model.
- Determination of the input random variables of the model.
- Model design.
- Determination of the probability density function of the input variables.
- Calculation of the probability density function of the output variable using Monte Carlo simulations.
- Estimation of parameters of the probability density function of the output variable and 95% coverage interval.

3. The Simulations of combined standard uncertainty in the thermovision measurements

The simulation research of the model (1) were carried out for different values of ε_{ob} and T_{ob} estimates. For every variant of simulations the combined standard uncertainty and 95% coverage interval were calculated. In the simulations the uniform distribution of input variables was assumed. In the Fig. 1, the probability density function of the output variable of described model was shown. The limits of the 95% coverage interval were marked with solid vertical lines. The histogram in Fig. 1 was obtained on the assumption that ε_{ob} = 0,8 and T_{ob} = 363 K. In this case the combined standard uncertainty yields 5,8 K (1,6%) and the minimal coverage interval is equal to $I_{95\%}$ = [354 374] K. Due to determination of the 95% coverage interval it is necessary to set the value of the α-quantile of the output variable. If the probability density function is symmetric, then the value of α can be expressed as:

$$\alpha = \frac{1-p}{2}, \tag{2}$$

where: *p* – level of confidence (for example, *p* = 0,95 (95%) corresponds to α = 0,025).

In this paper the two 95% coverage intervals were compared. The first was obtained from the distribution function of the output variable of the model (1). The second one was calculated considering, that the output variable has a normal distribution. The relationship between the α parameter and the 95% coverage interval was presented in the Fig. 2. The 95% coverage interval obtained from Monte Carlo simulations was marked with solid line. The 95% coverage interval considering normal distribution was marked with broken line.

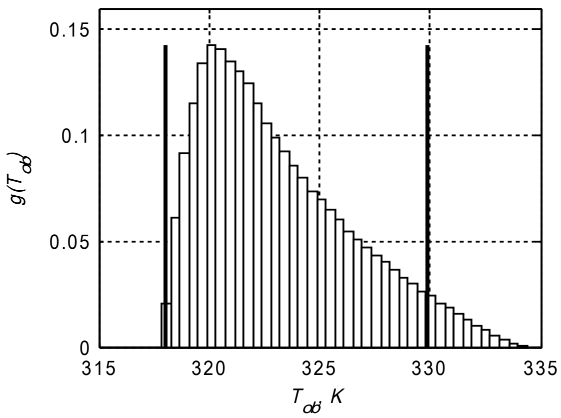

Fig. 1. *The probability density function of the output variable in the model (1) for T_{ob} = 323K and ε_{ob} = 0,8*

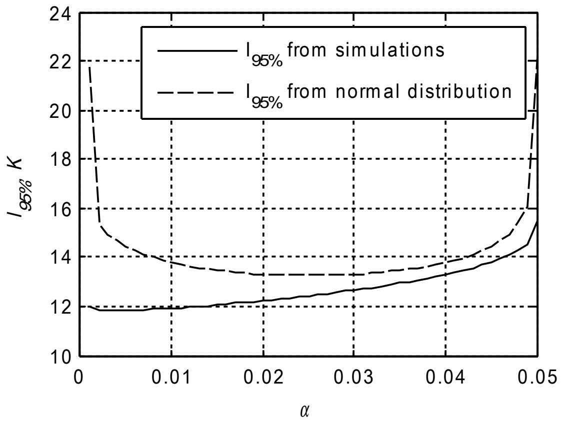

Fig. 2. *The 95% coverage interval as the function of α parameter for T_{ob} = 323K and ε_{ob} = 0,8*

4. Conclusions

- The value of combined standard uncertainty strongly increases with the decrease of the object emissivity ε_{ob}.
- Comparing the 95% coverage intervals from approximation of distribution function of output variable and from the assumption of normal distribution, one can observe that the differences are negligible.
- The assumption of the normal distribution of the output variable of the model (1) – according to the central limit theorem – is safe from the viewpoint of the underflow of 95% coverage interval.

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